

Dépliage de polycube de genre élevé



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DGMM Nov 2021



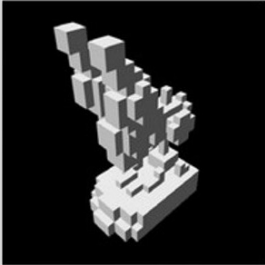


Plan

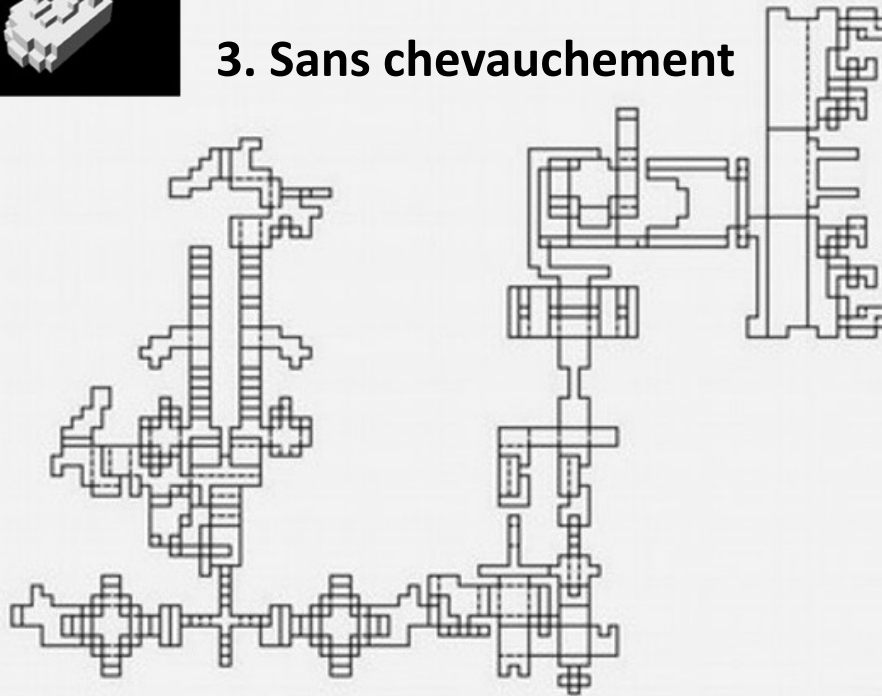
1. Contexte et définitions
2. Polycubes de Menger
3. Eponges de Menger

Dépliage de Polycubes :

De l'Art à la Science



1. Représentation planaire
2. Connecté
3. Sans chevauchement



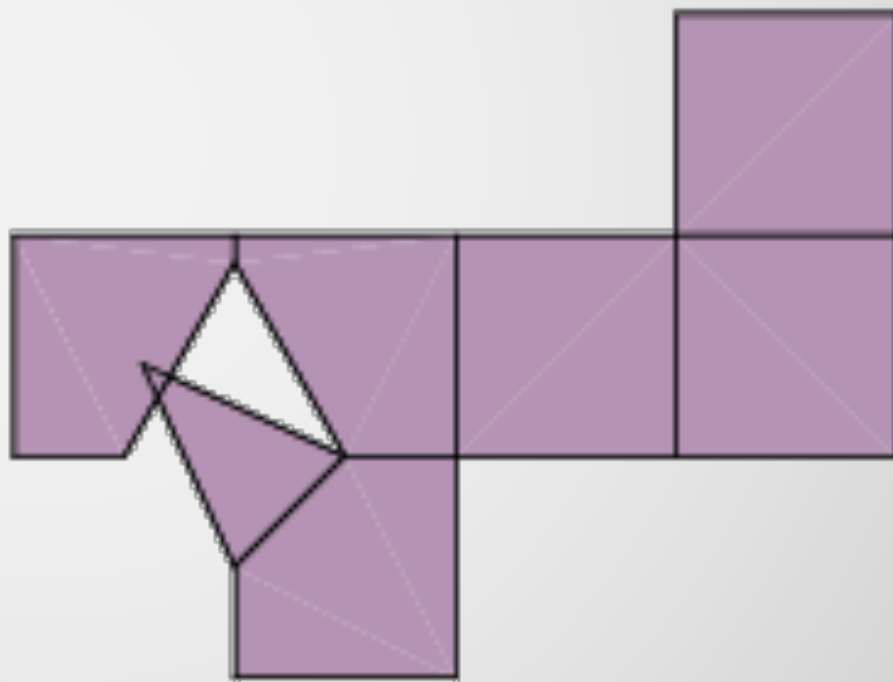
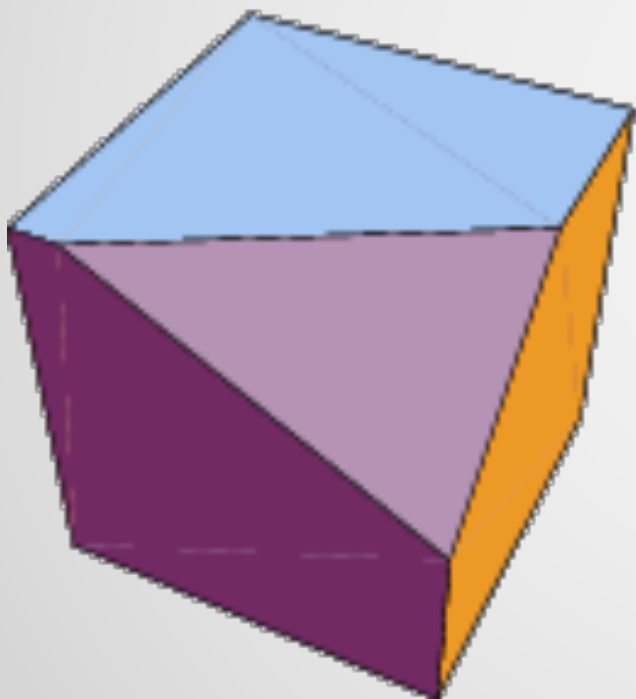
Aurélié Mourier, Stéréomicroscope et son déployé –
surface 00797.001 Maille plastique cousue
150 x 90 x 250 cm, 2011, production l'Aparté



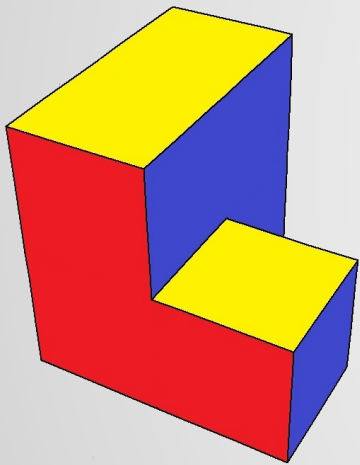
Stereo-microscope,
Aurélié Mourier

Certains objets ne sont pas dépliables

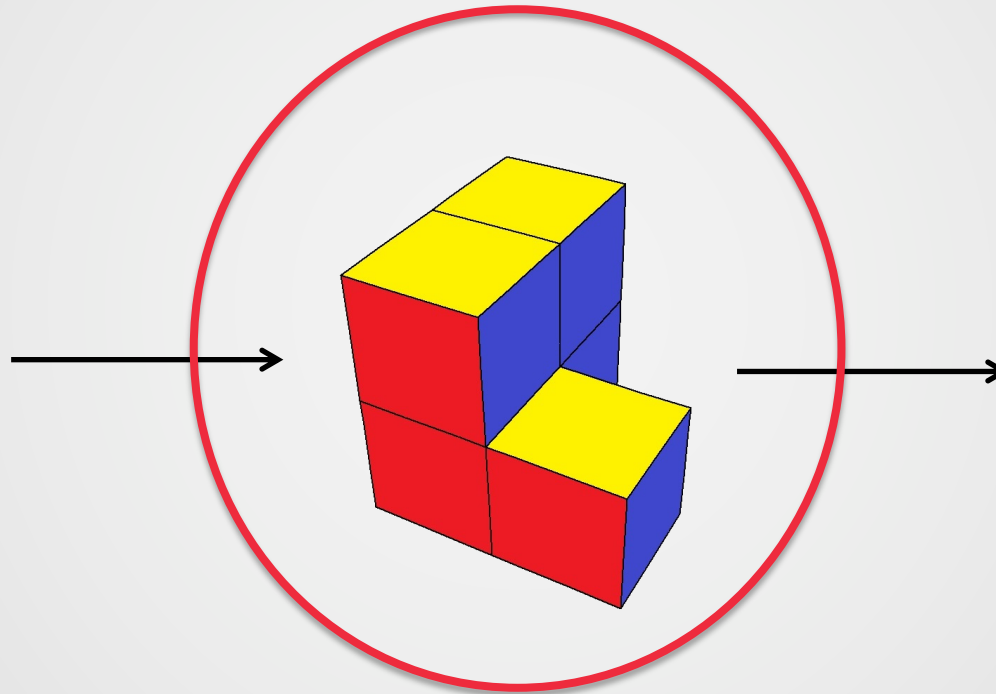
Contre-Exemple pour les surfaces triangulées



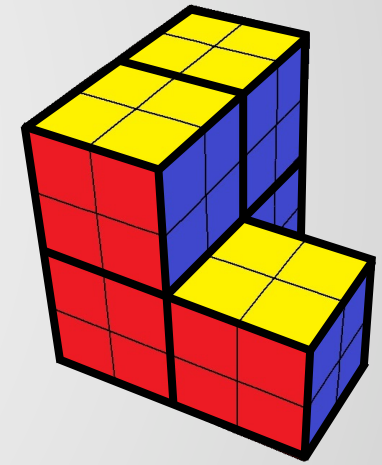
Dépliage par arêtes ou par grille



Dépliage par arêtes



Dépliage par grille



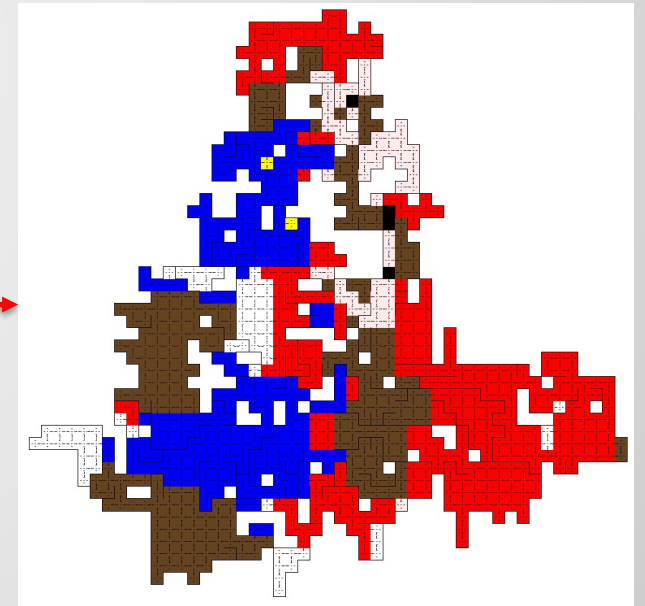
Dépliage par grille
avec raffinement

Déplier les polycubes sans raffinement est-il toujours possible ?



Polycube

Algorithme, dénombrement,
contre-exemple ?

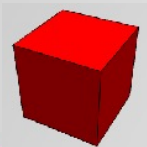


Patron ?

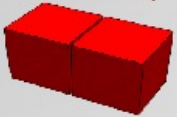


Combinatoire

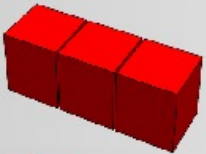
- Dénombrement de patrons ?



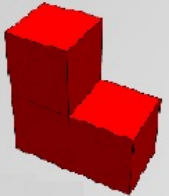
11



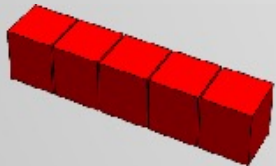
792



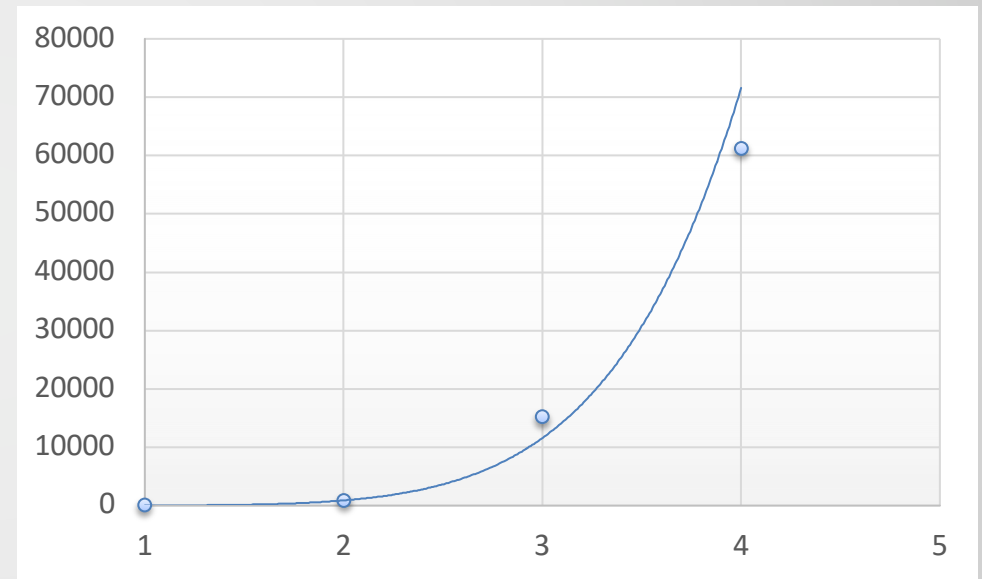
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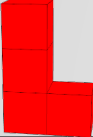
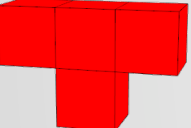
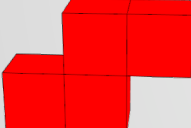
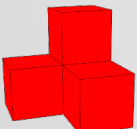
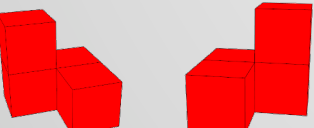
131 966



1 099 511 627 776



Nombre de patrons selon l'agencement

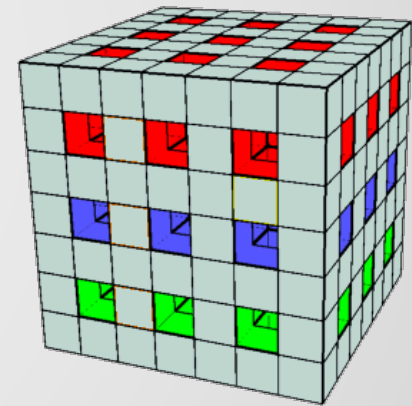
Polycube	Arbres couvrants	Arbres dépliés	Patrons distincts	Degrés des sommets
	207.746.836	207.735.220 (99%)	61,186	D3 : 8 D4 : 12
	201.727.636	102.267.402 (51%)	1,137,494	D3 : 10, D4 : 8, D5 : 2
	196.230.936	63.052.400 (32%)	983,401	D3 : 12, D4 : 4, D5 : 4
	194.907.648	66.424.010 (34%)	1,140,873	D3 : 12, D4 : 4, D5 : 4
	189.803.898	54.762.972 (28%)	913,932	D3 : 13, D4 : 3, D5 : 3, D6 : 1
	195.375.936	75.587.600 (38%)	2,101,254	D3 : 12, D4 : 5, D5 : 2, D6 : 1

D'après le mail de Donald Knuth à George Miller



Plan

1. Contexte et définitions
2. Polycubes de Menger
3. Eponges de Menger





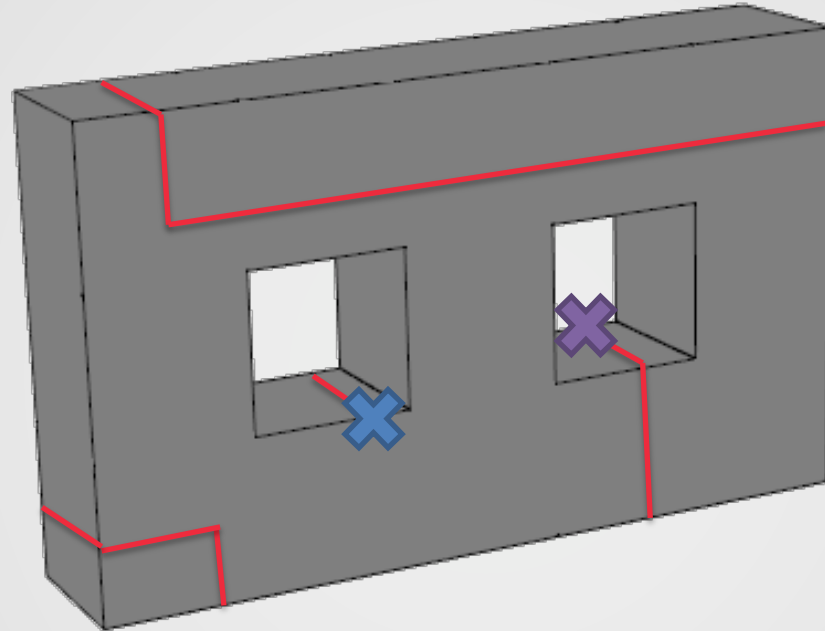
Biblio genre > 2

- Orthographe (1x2) (Ho 2017)
- Polyèdres orthogonaux avec 2 directions de trous (1x2) (Ho 2017)
- Polyèdres à une couche de genre non contraint (1x2) (Chang 2017)

Toujours du raffinement : chaque trou est parcouru 2 fois par le dépliement.



Cas du genre 2

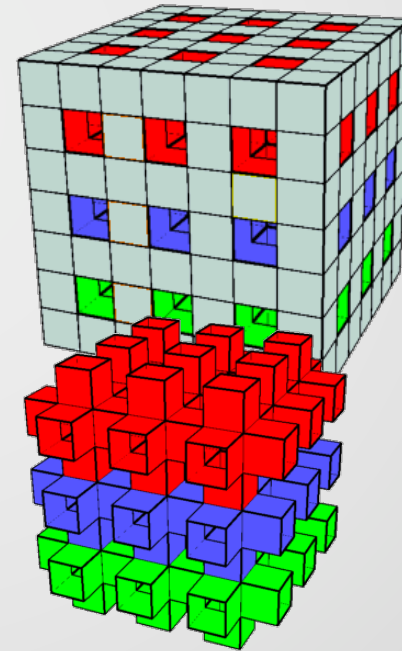
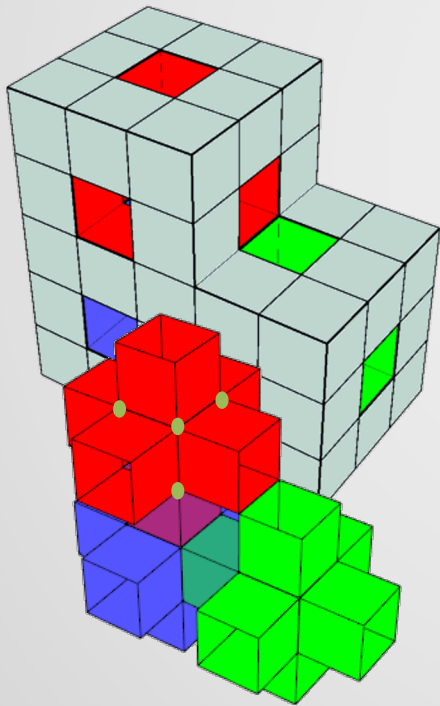


Damian (2017)
Raffinement ($O(n) \times O(n)$)

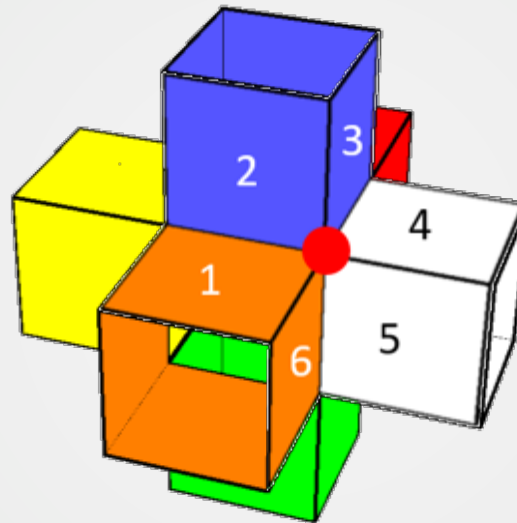
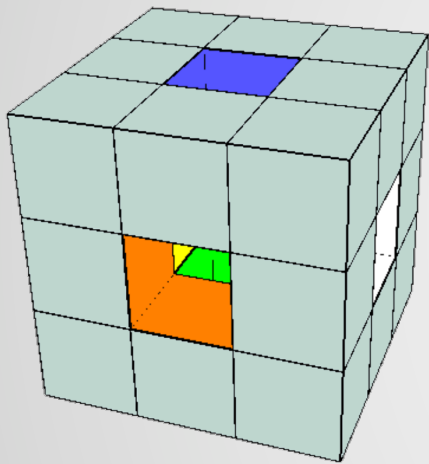




Polycubes de Menger



Sommets de degré 6

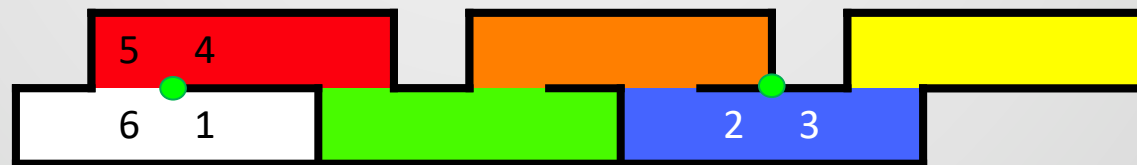
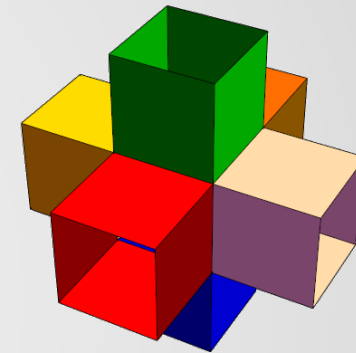
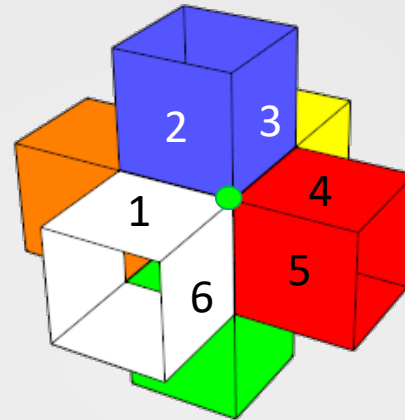
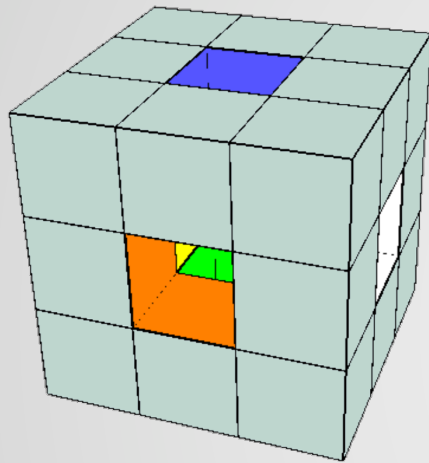


2	3
X	4

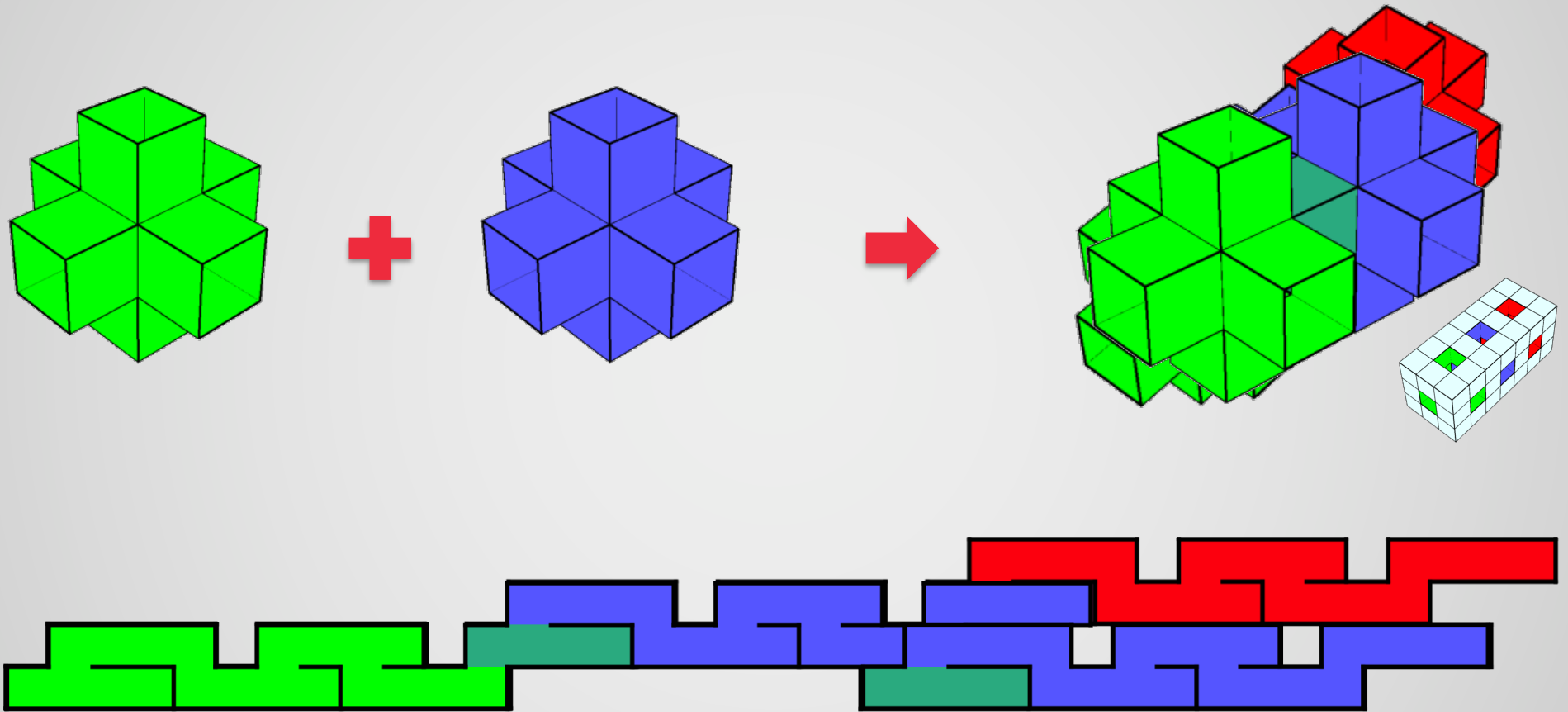
Les sommets de degrés six ne sont pas dépliables seuls.



Déplier une croix

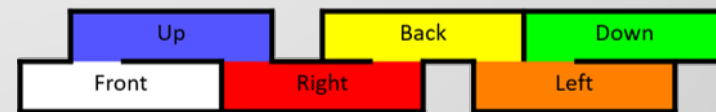
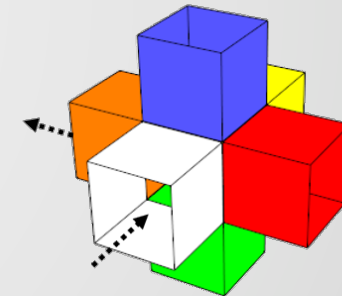
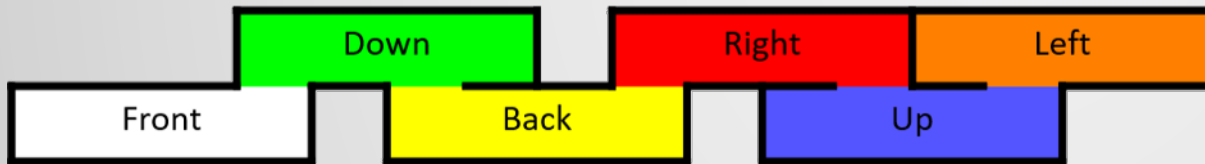
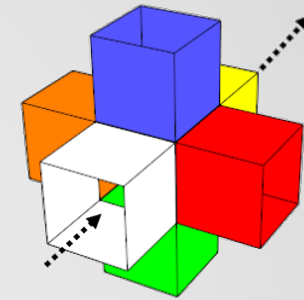
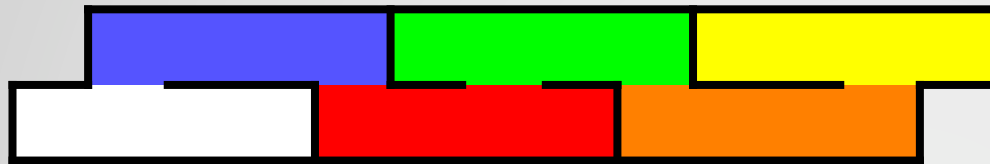


Déplier une ligne de croix





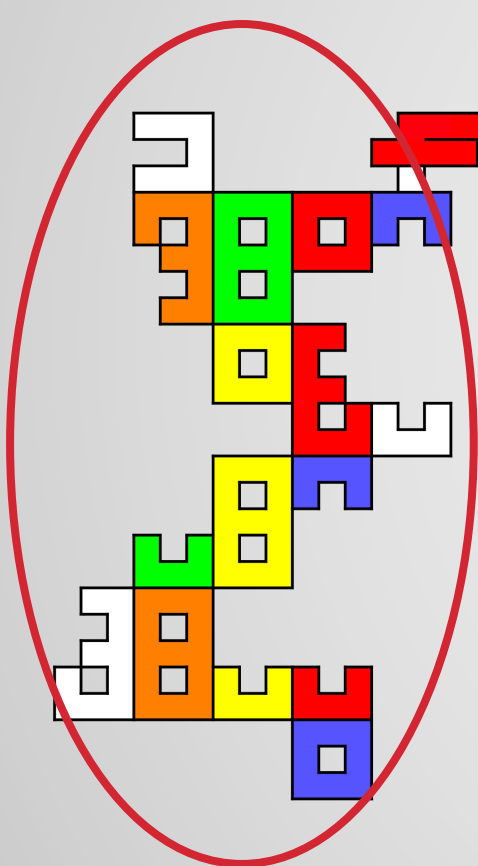
Différentes configurations



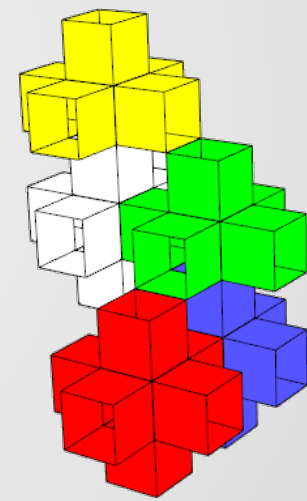
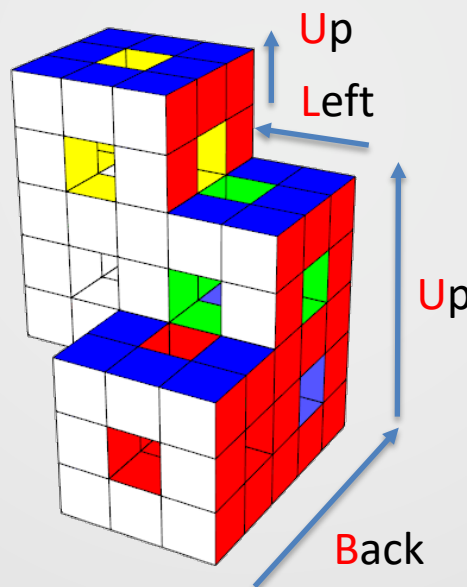
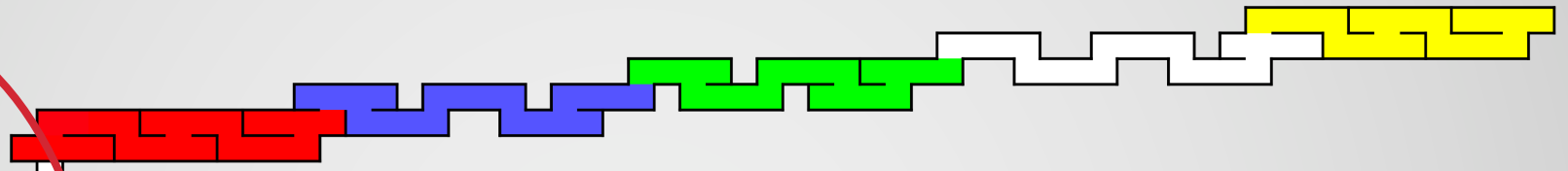


Tubes de Menger

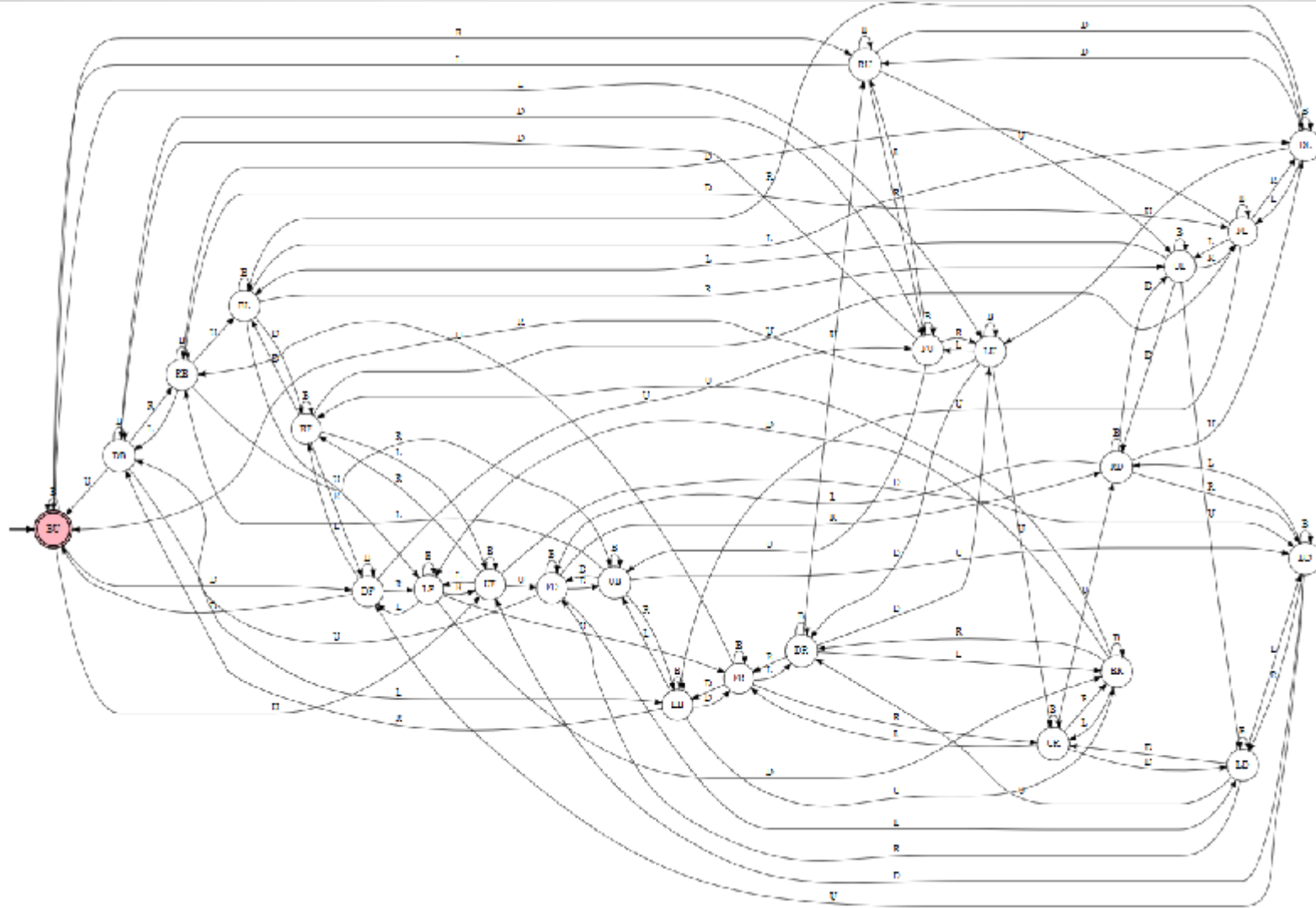
BULU → ANWEA



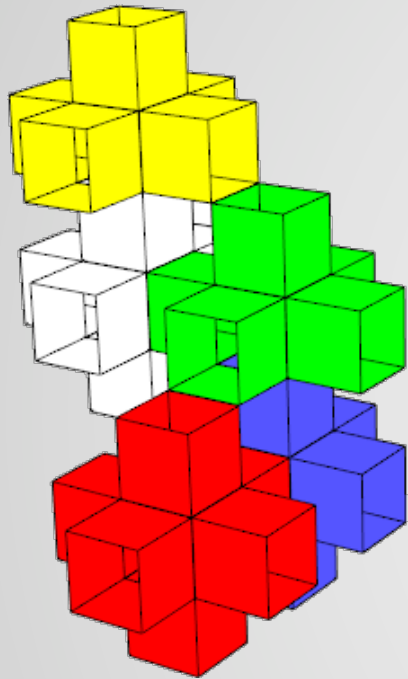
Biedl 98



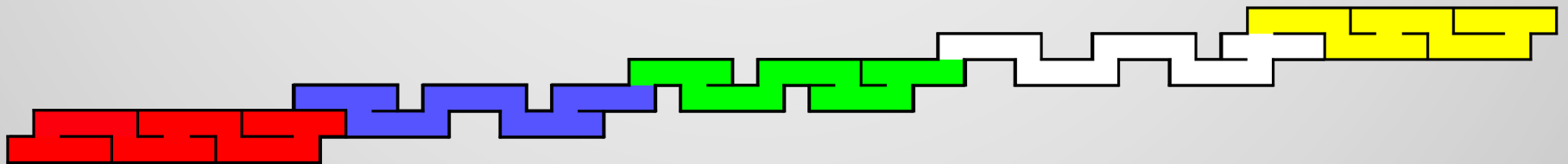
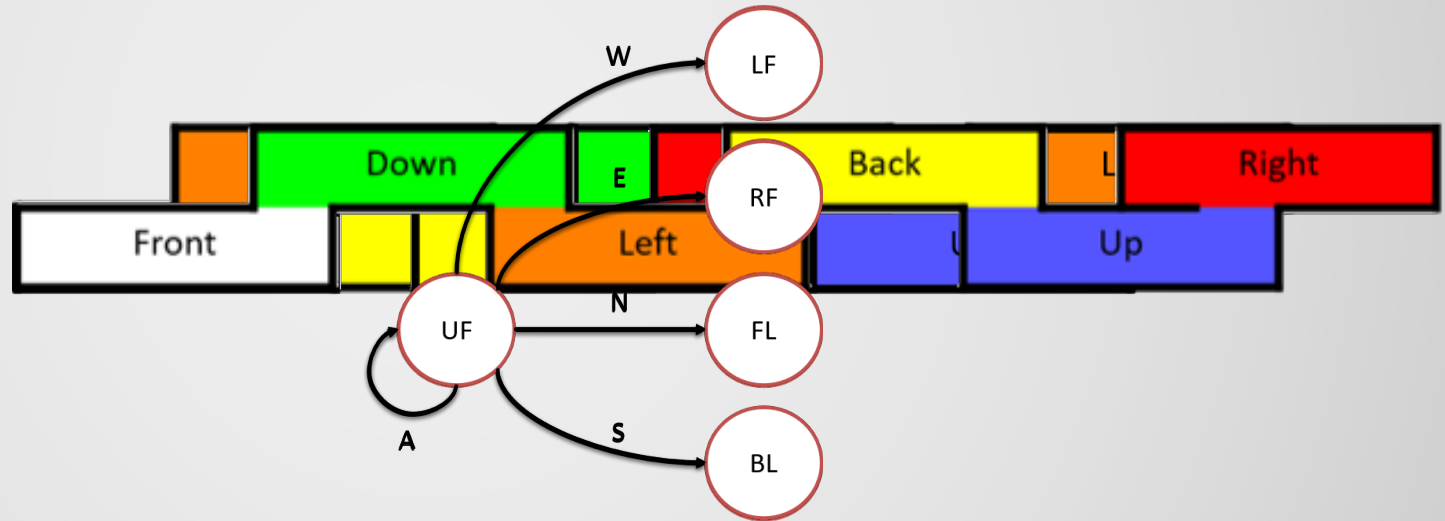
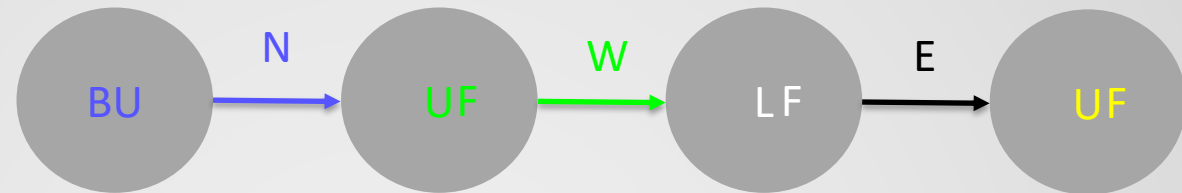
Automate



Patron des faces intérieures



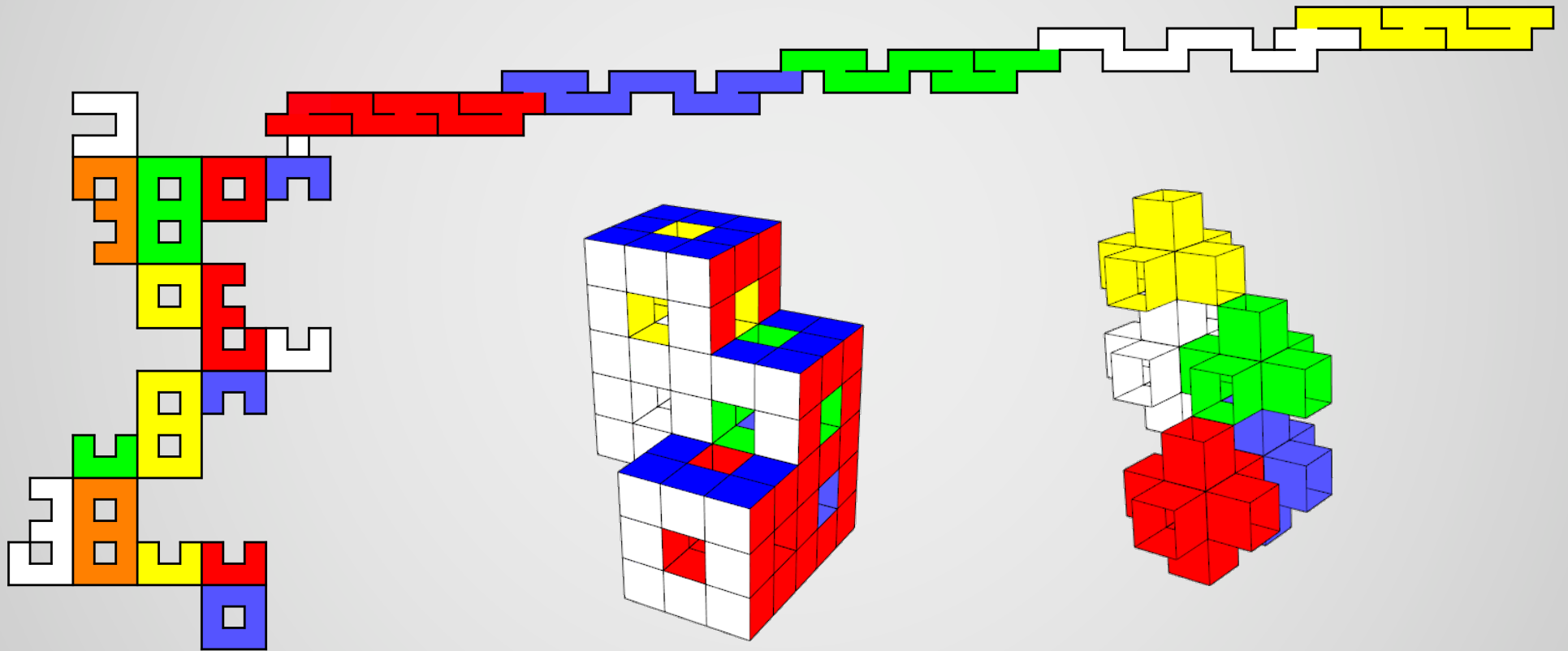
BULU



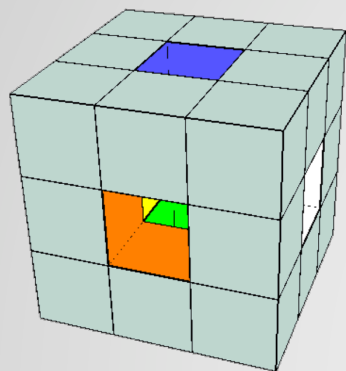


Résultat final

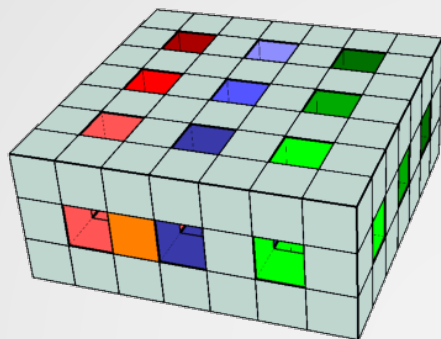
BULU → ANWEA



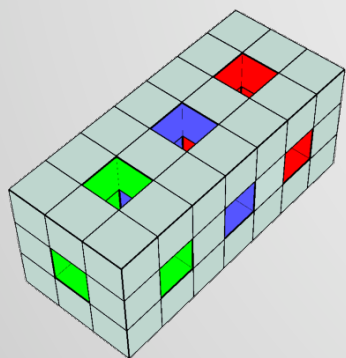
Polycubes de Menger Parallélèpipèdes



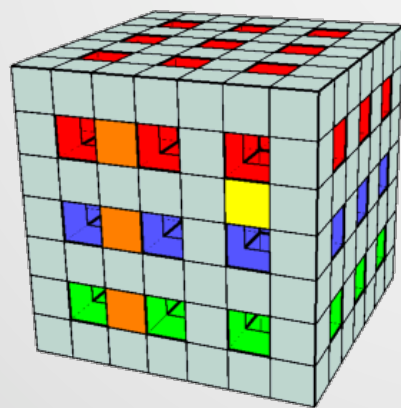
Genre 5



Genre 33



Genre 13

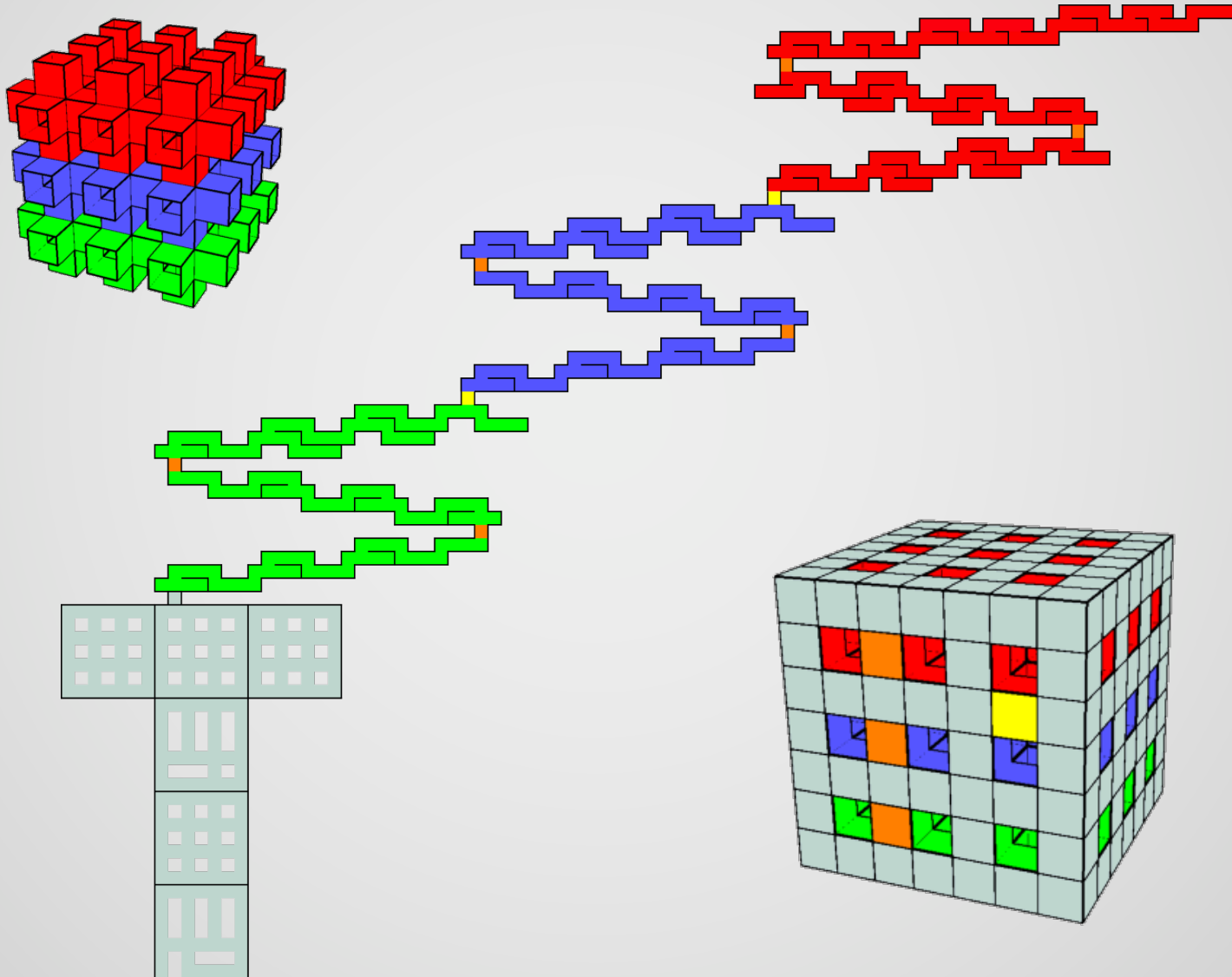


Genre 81

- Nombreux sommets de degré 6
- Genre élevé

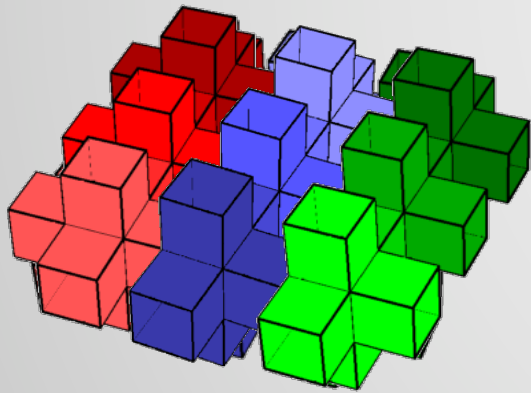
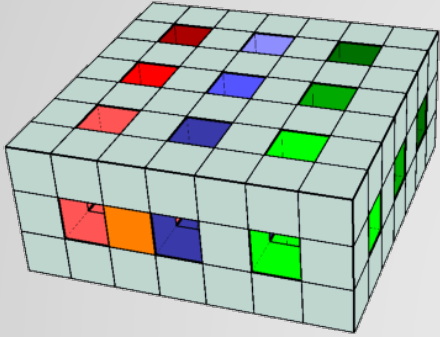


Aperçu





Déplier un plan



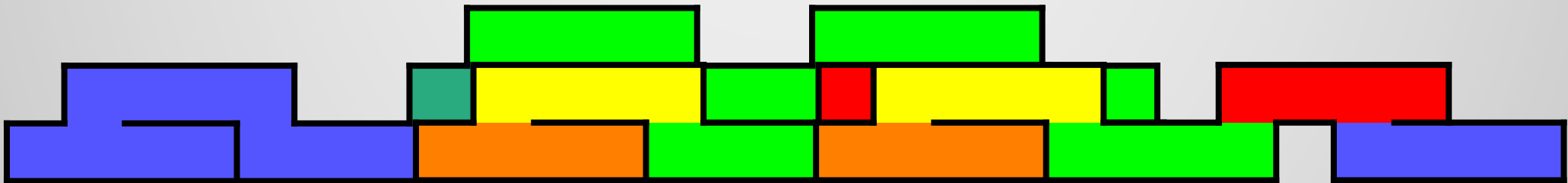
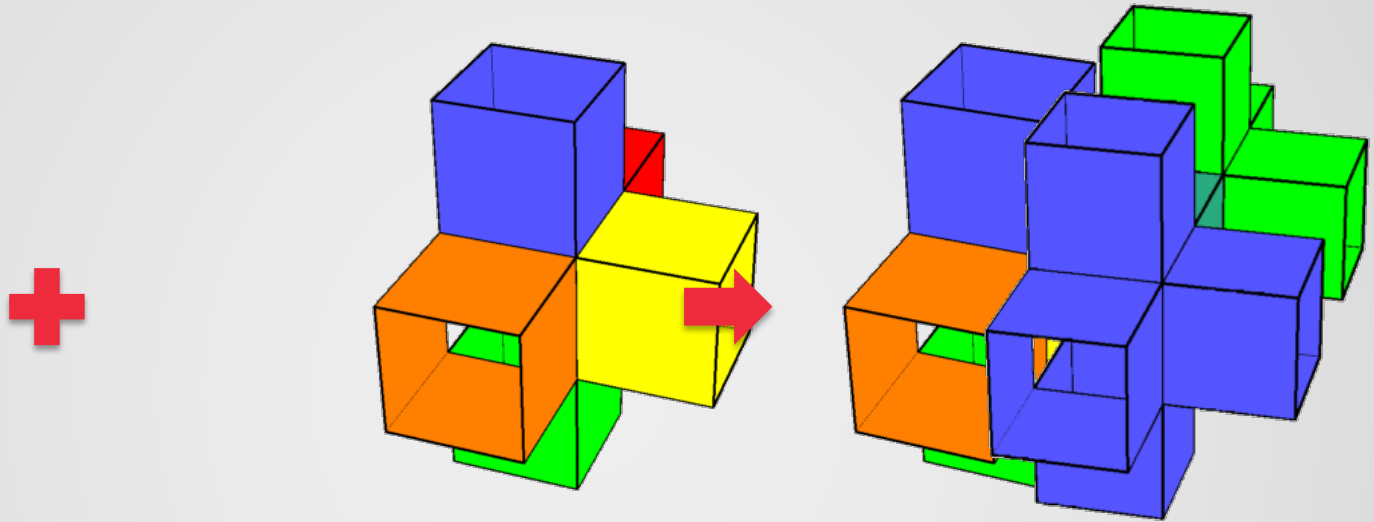
Plan

Ligne Complète

Lignes partielles

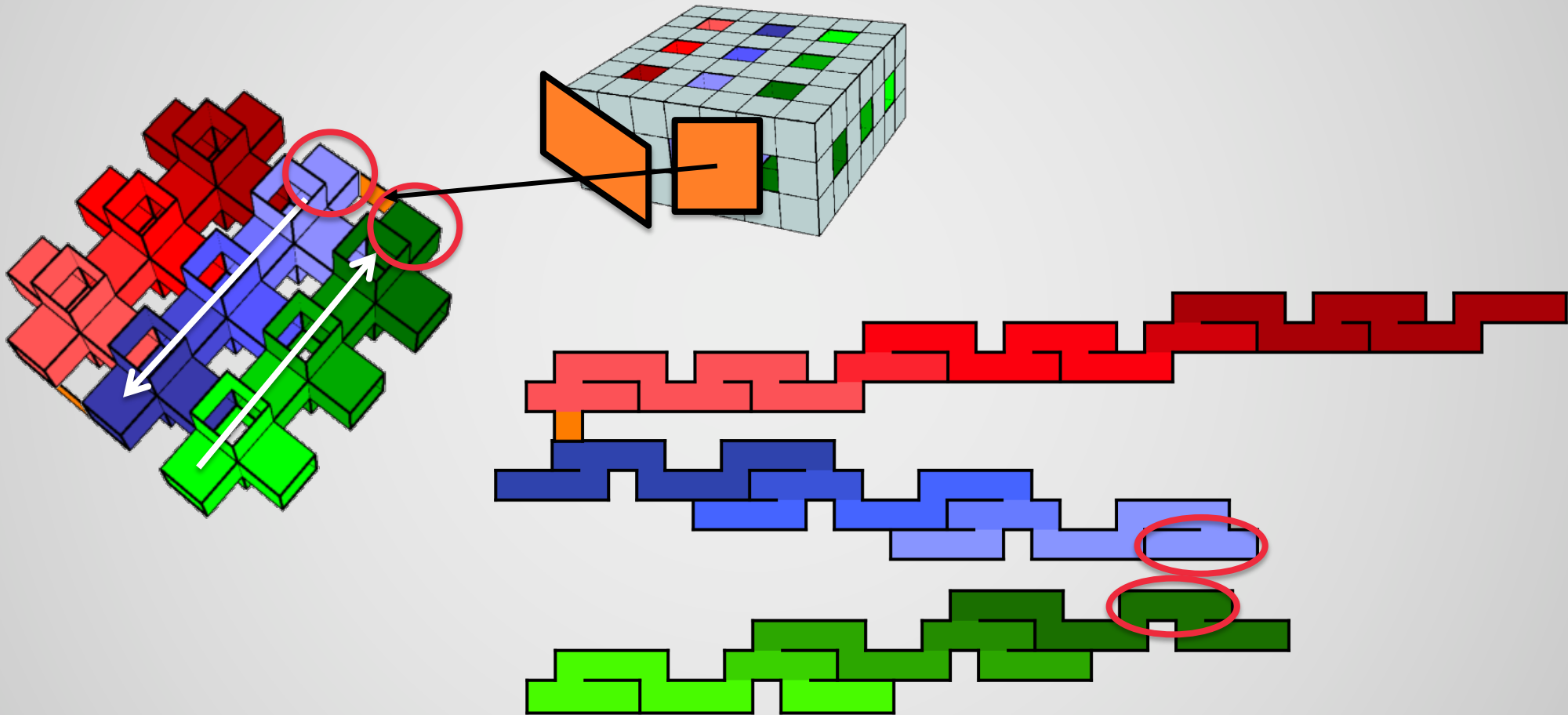


Déplier un plan: les lignes partielles



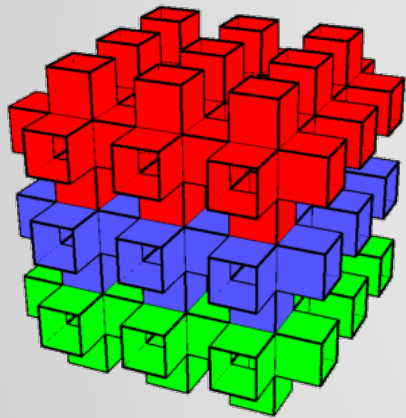


Déplier un plan: connecter les lignes



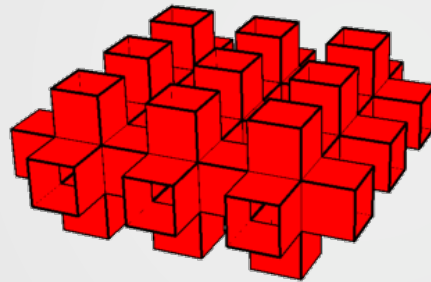


Déplier un volume



Volume

=

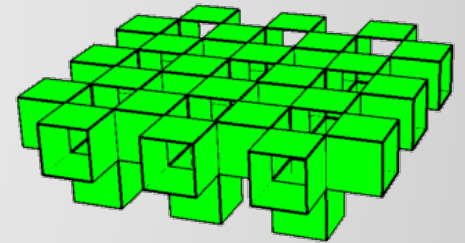


Plan complet

+

2

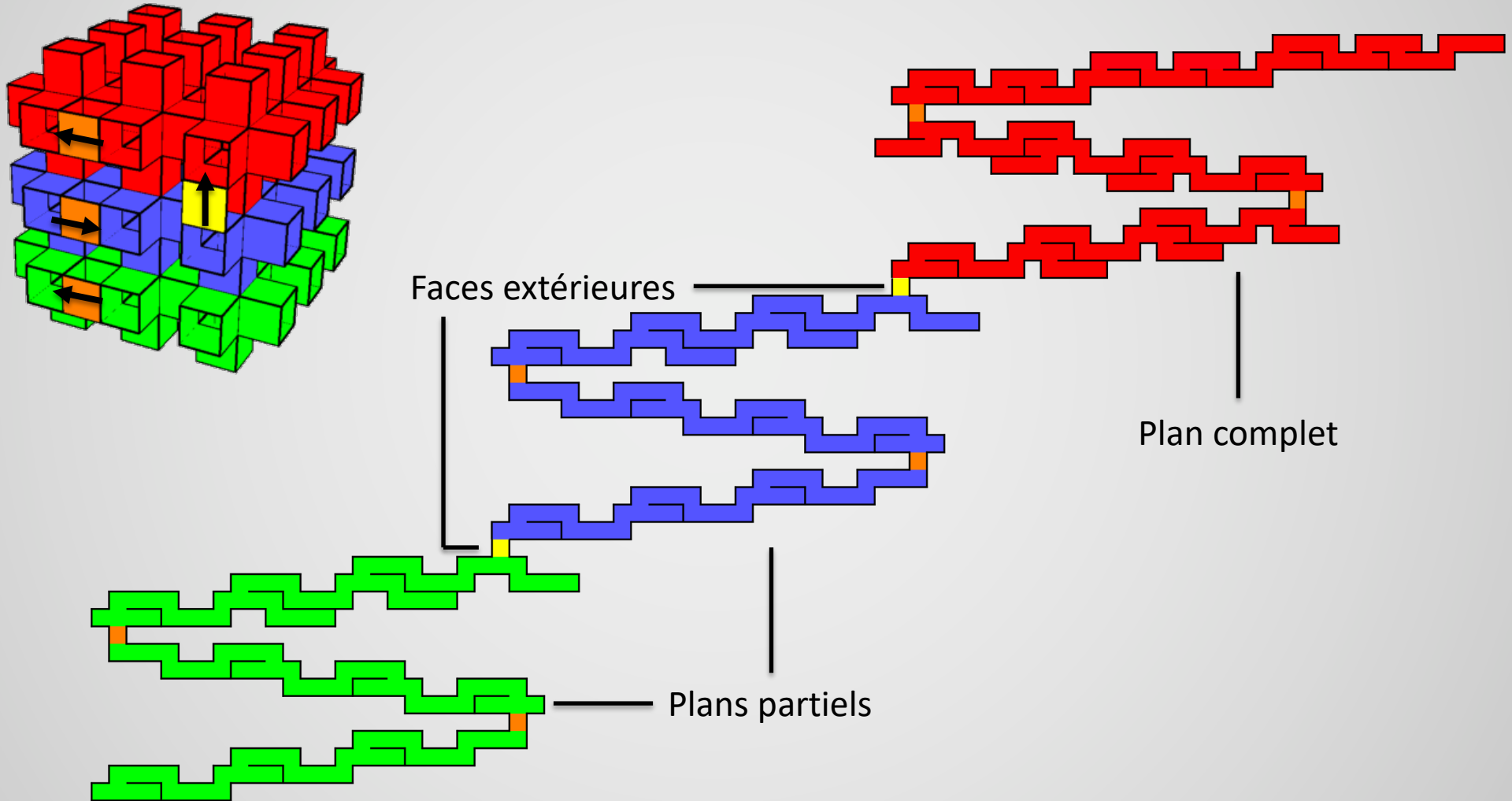
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Plan Partiel

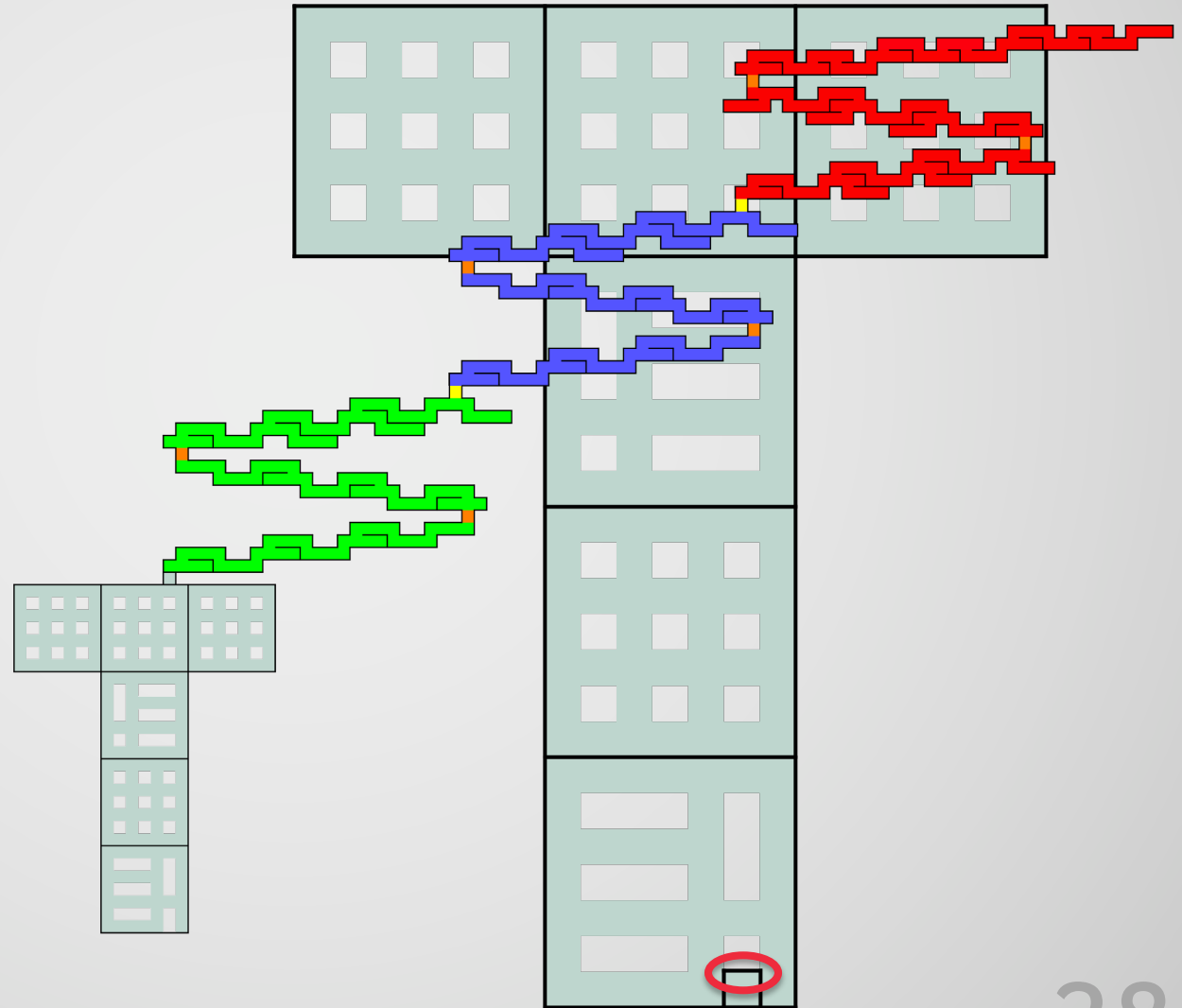
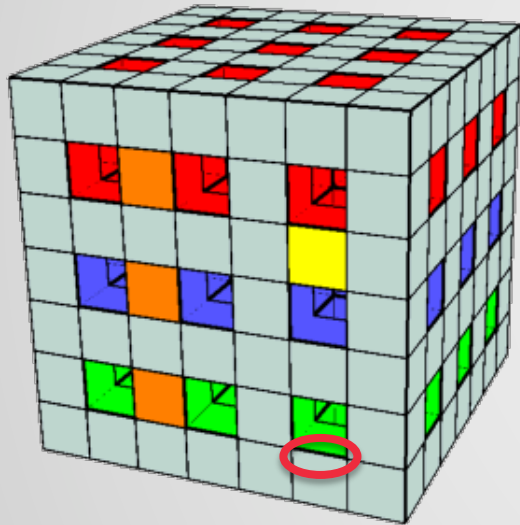


Déplier un volume: connecter les dépliements de plans



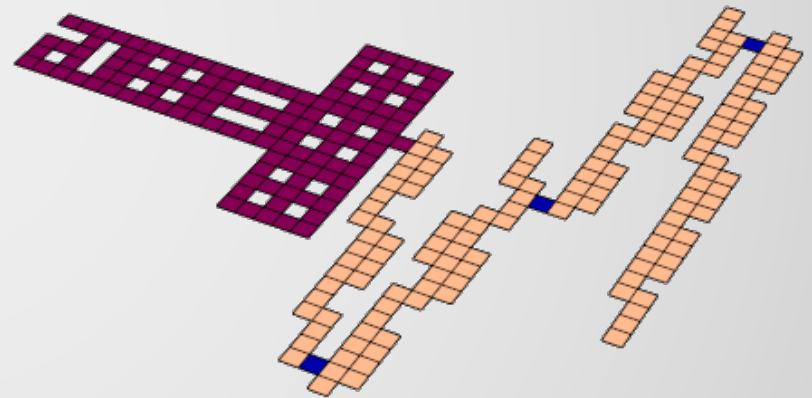


Déplier les faces extérieures



Polycubes de Menger : Conclusion

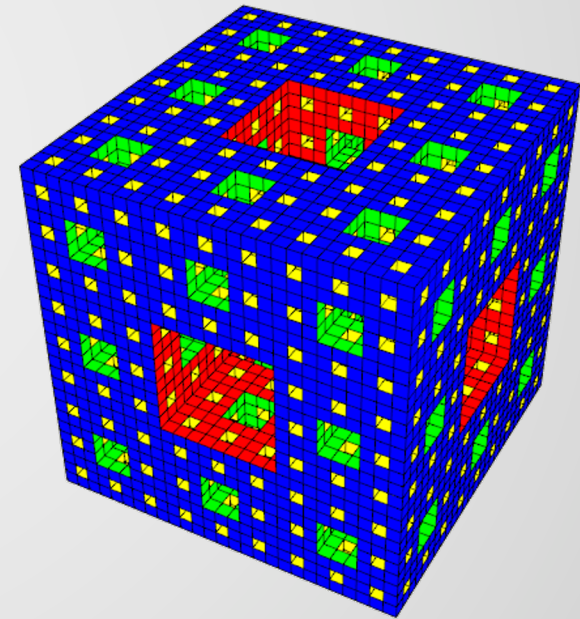
- Algorithmes déterministes pour les tubes de Menger et les parallélépipèdes de Menger
- Parcours linéaire de la structure interne



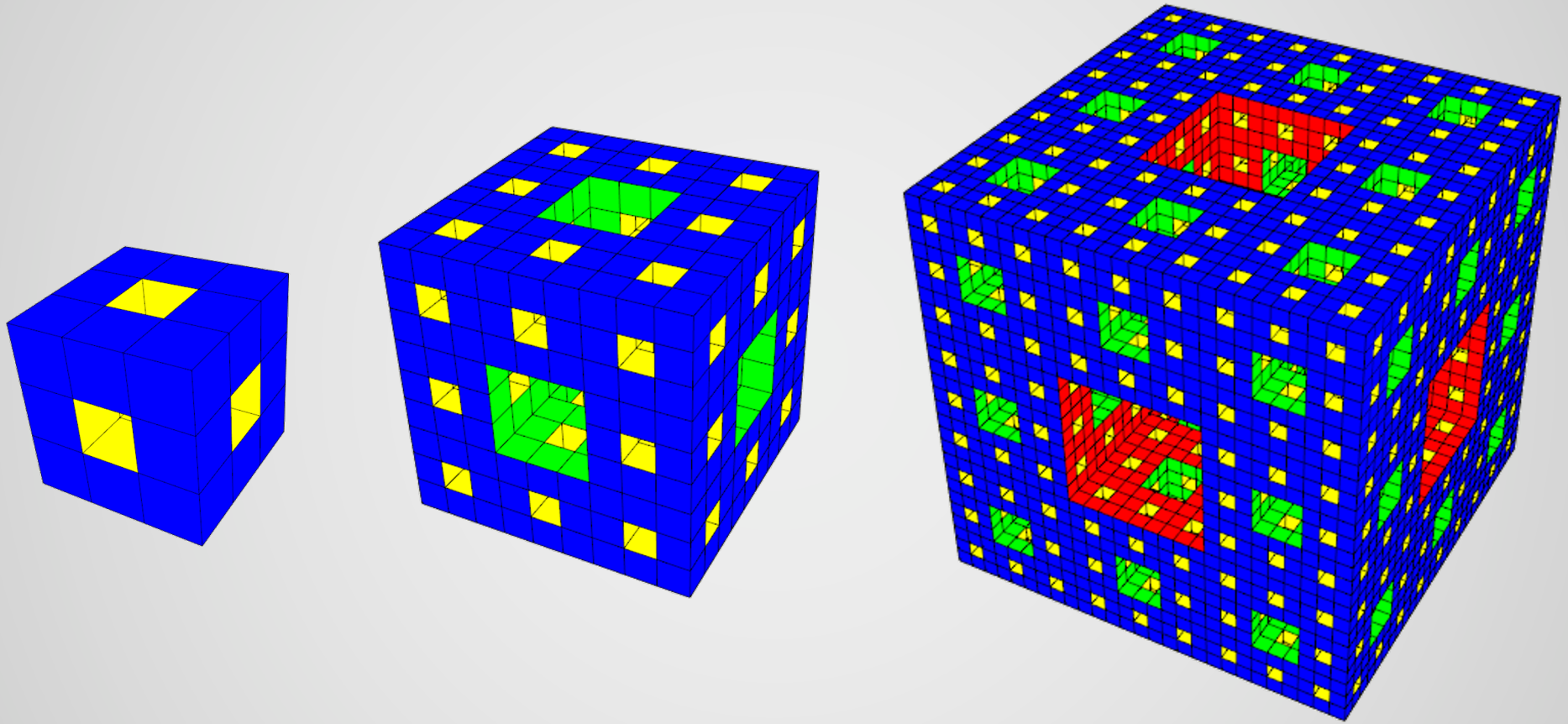


Plan

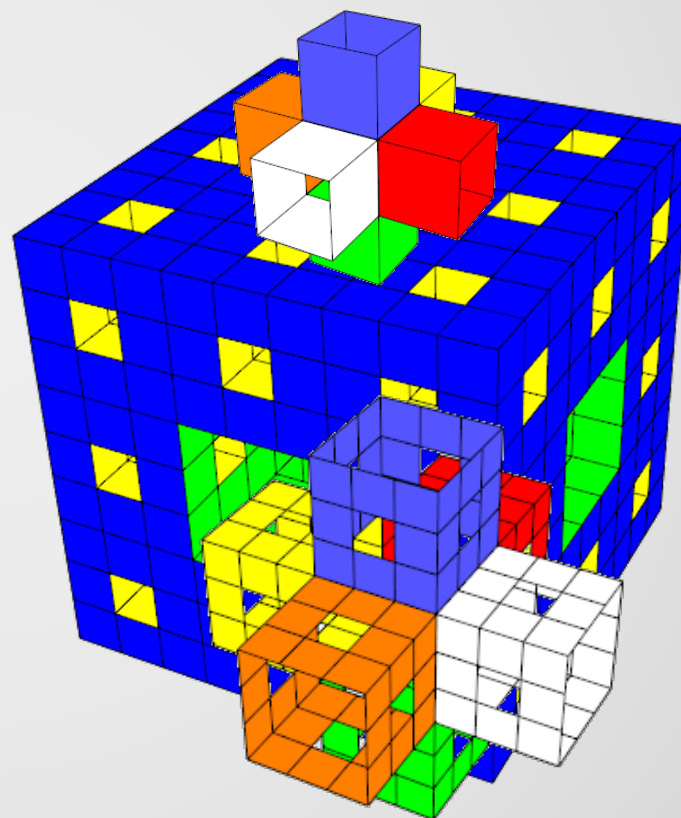
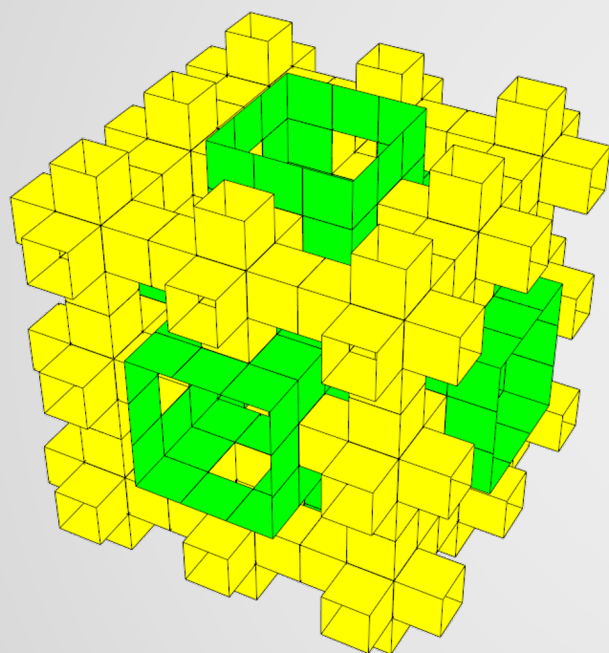
1. Contexte et définitions
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Eponges de Menger

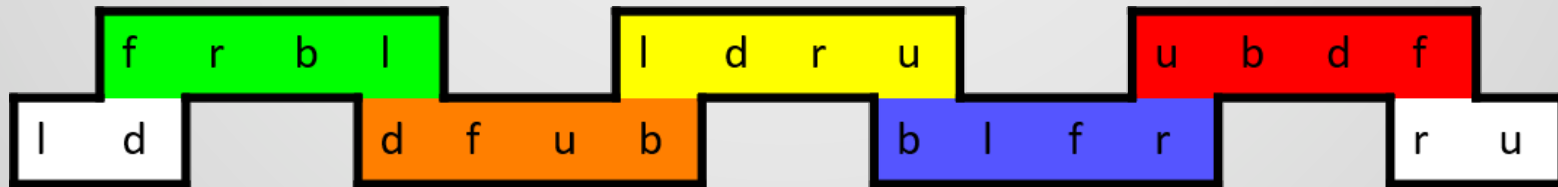
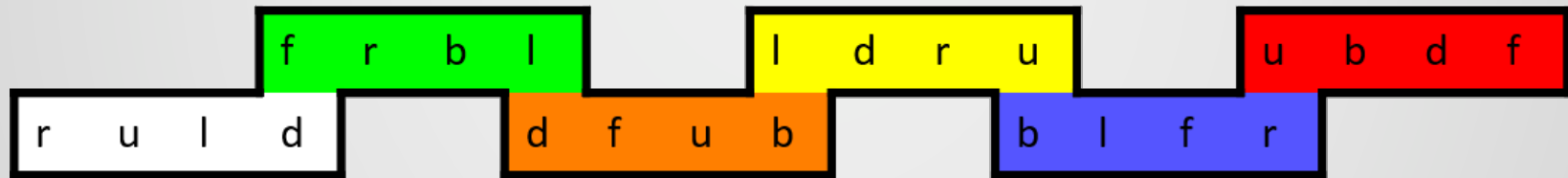
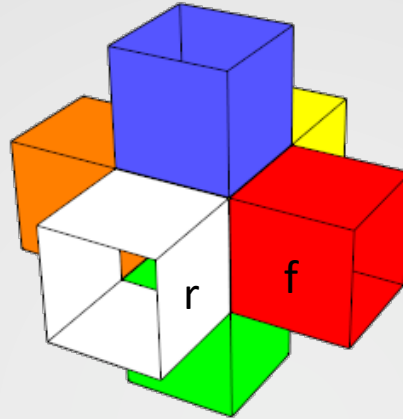


Structure d'une éponge de Menger



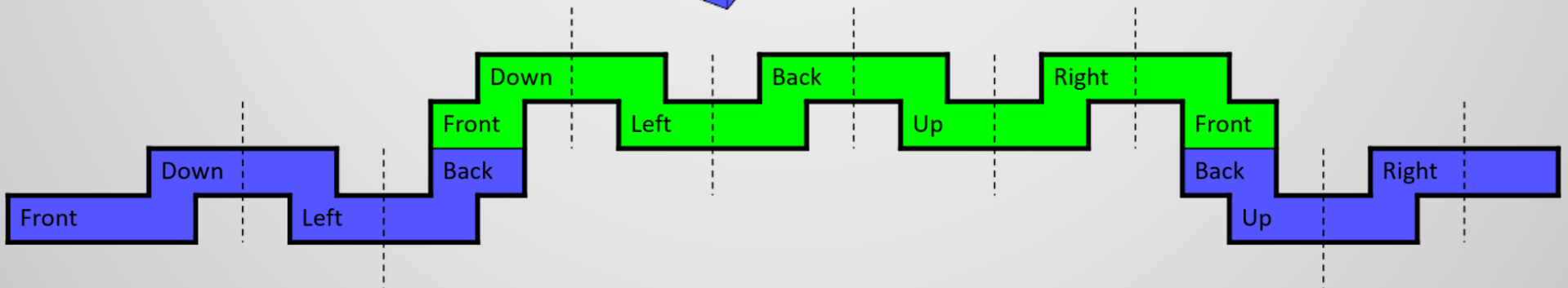
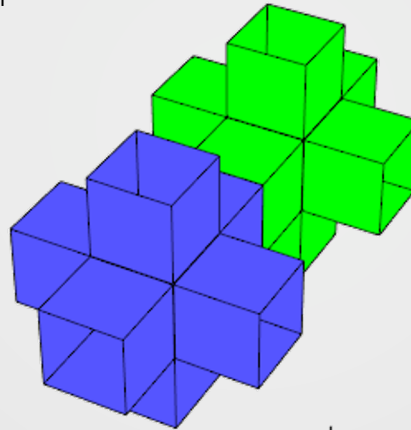
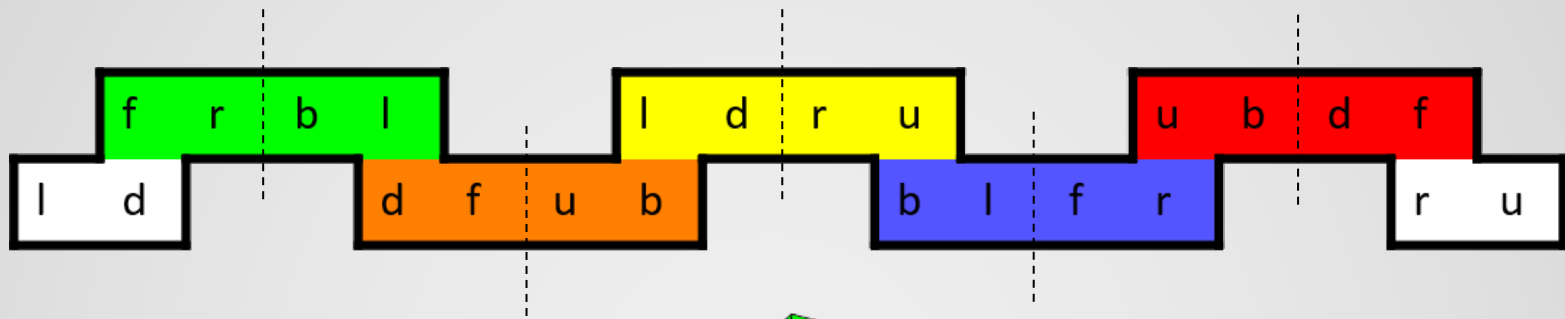


Déplier une croix



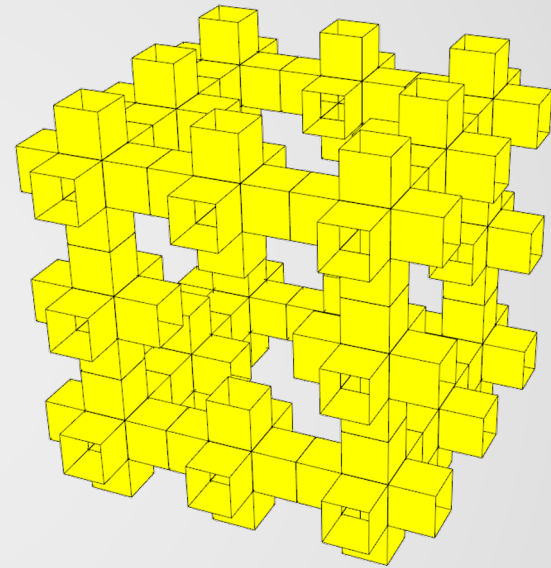
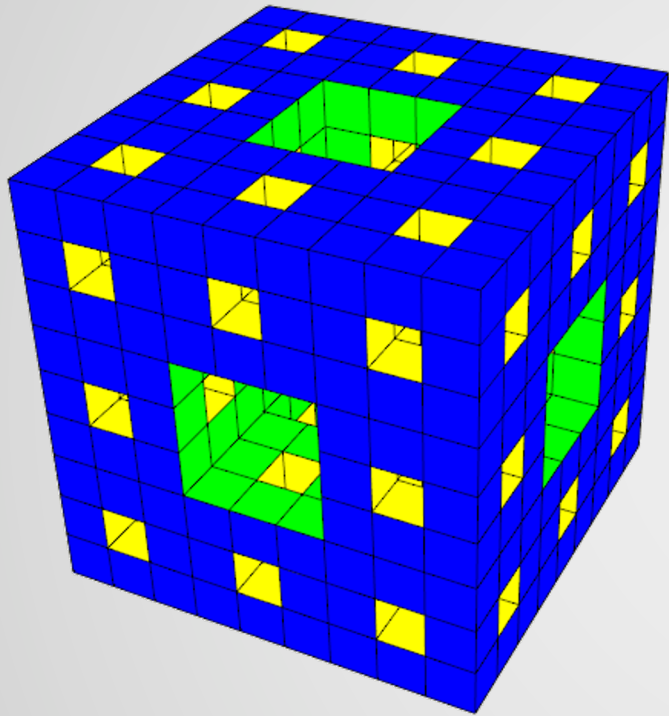


Déplier une suite de croix



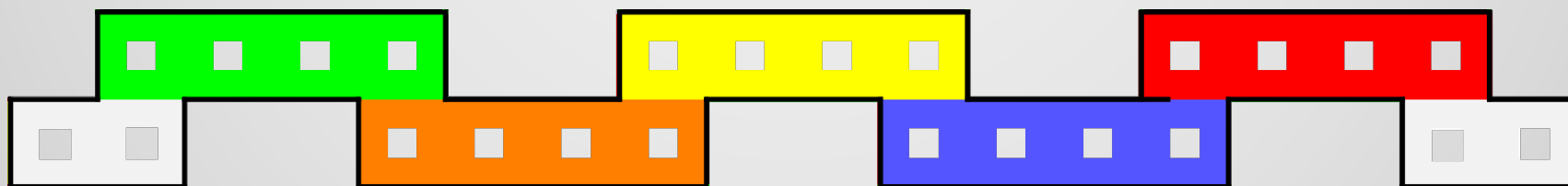
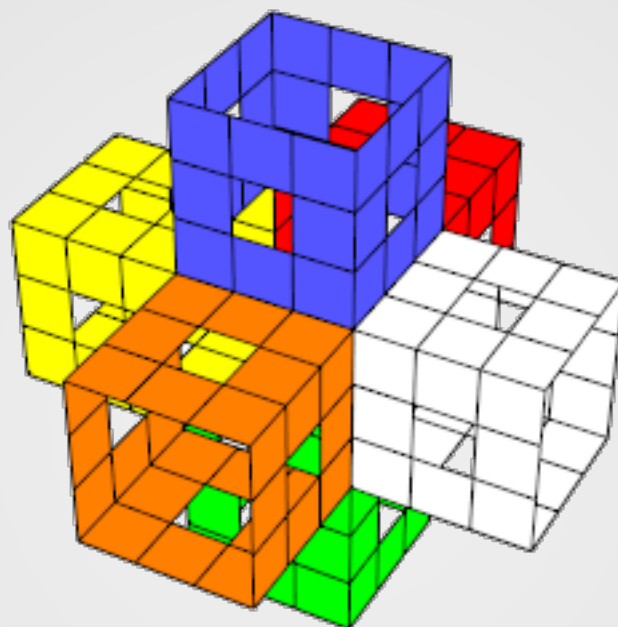


Dépliage d'un niveau de croix



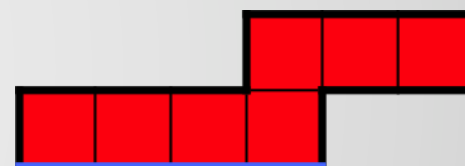
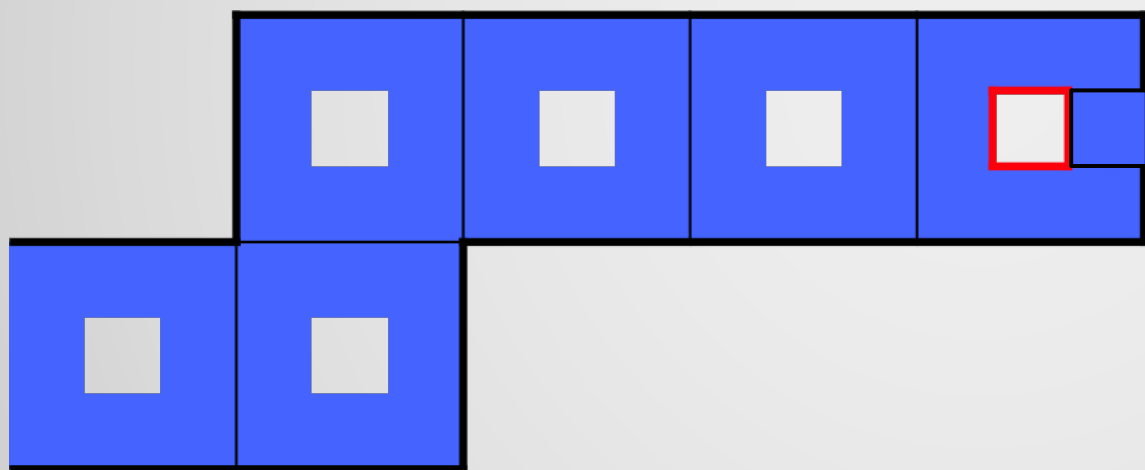


Déplier une croix de niveau 2 ou plus



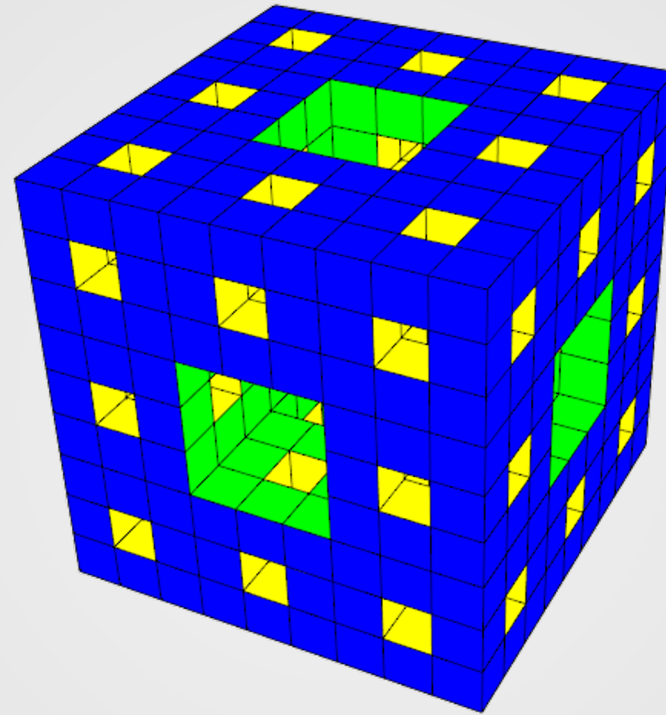


Relier les patrons de 2 niveaux





Résultat final





Conclusion

- Algorithme déterministe pour déplier les éponges de Menger de niveau quelconque
- Calque la structure récursive de l'objet
- Peut être étendu à des assemblages d'éponges

Merci pour votre attention

<https://www.youtube.com/watch?v=5Cj5520oZVI>

