Journées du GT Géométrie Discrète et Morphologie Mathématique

Statistical modeling of pulmonary vasculatures with topological priors in CT volumes

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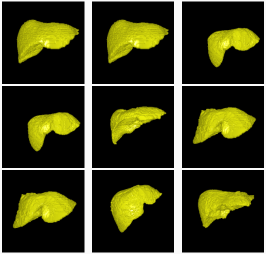
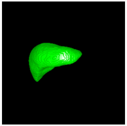
Statistical anatomical models

Statistical models

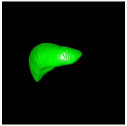
- represent organ shapes and intensity distributions by a few number of parameters;
- have important roles in medical image analysis, for example,
  - segmentation,
  - super-resolution,
  - modality transform, etc.



Liver shape variations



Model 1

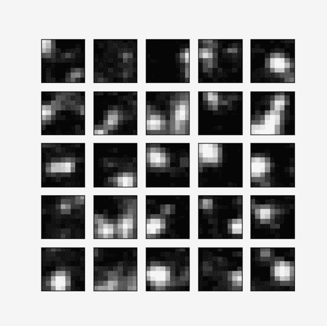


Model 2

Statistical modeling of blood vessels

- **Less study on modeling blood vessels** has been made, compared to solid organs such as livers, as the shapes and the intensity distributions are highly complex.
- There are **more variations** with respect to the intensities, directions, sizes, etc.

Pulmonary CT image



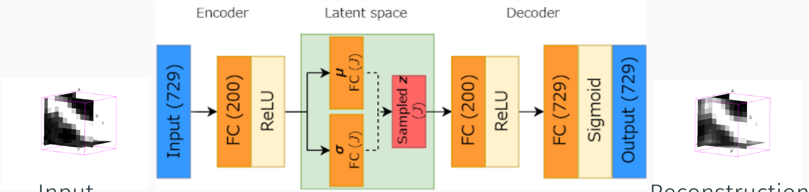
3D volume patches ($9 \times 9 \times 9$)

Various studies on modeling of blood vessels

1. **Geometrical profile model**
   - based on cylinder whose cross sections are ellipses
2. **Statistical intensity model**
   - Principal component analysis (PCA)
     - incompatible with non-linear distributions
   - Manifold learning
     - incompatible with complex distributions
   - Deep learning
     - Stacked AudoEncoder (SAE)
     - Variational AutoEncoders (VAE)

VAE (previous work) and the problem

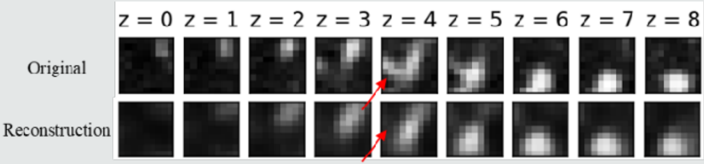
Volume patches containing a blood vessel are modeled based on VAEs.



Input Reconstruction

Problem

The model fails to represent anatomical features of blood vessels: **missing a bifurcation**.



Loss function of β-VAE and its limitation

Given a volume patch $\mathsf{x}^{(i)}$ and its reconstruction $\mathsf{y}^{(i)}$, the loss function of β-VAE is defined by

$$\mathcal{L}_{\beta\text{-VAE}} = -\frac{\beta}{2}\sum_{j=1}^{J}\left(1+\log((\sigma_j^{(i)})^2)-(\mu_j^{(i)})^2-(\sigma_j^{(i)})^2\right)+\frac{1}{2}\|\mathsf{x}^{(i)}-\mathsf{y}^{(i)}\|_2^2$$

where $\mu^{(i)}$ and $\sigma^{(i)}$ are the mean and variance of the latent space variables, computed from the training data and $\mathsf{x}^{(i)}$.

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1. **Regularization**: approximated by Gaussian distribution

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1. **Regularization**: approximated by Gaussian distribution
2. **Reconstruction error**

Problem

Impossible to consider global topological features!

Objective

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Building a **statistical intensity model of pulmonary vasculatures** in CT volume patches that incorporates **topological priors**.

More precisely, topologically correct modeling of blood vessels, which allows to represent **bifurcations**.

Proposed method

Summary:

- Use a conventional network of β-VAE (Saeki et al., 2019), whose loss function is

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- Design the model such that the reconstructions meet topological priors, whose **loss function** is defined by

$$\mathcal{L} = \mathcal{L}_{\beta\text{-VAE}} + \mathcal{L}_{\text{topo}}$$

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Pre-training

- **Topological Loss** (Clough et al., 2020) is used for topology of blood vessels.
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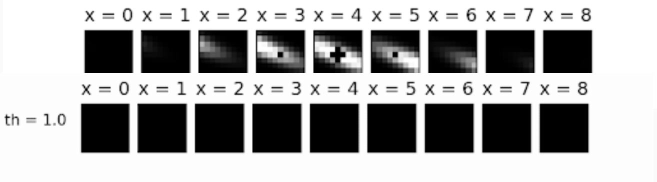
Fine tuning

Persistent Homology (PH)

PH enables to quantify topological features of gray images.

- Gray image and its thresholded images



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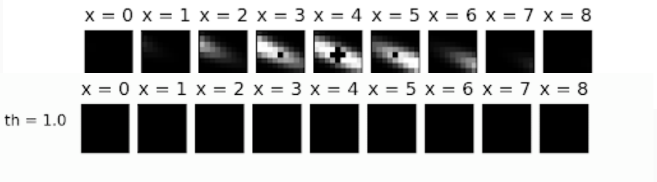
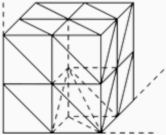


- Convert a gray image to a simplicial (cubical) complex.



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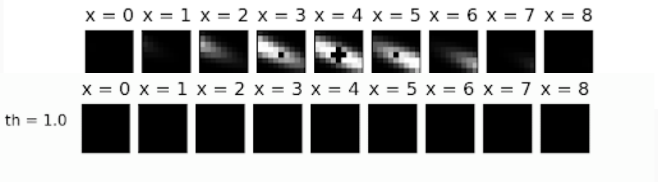
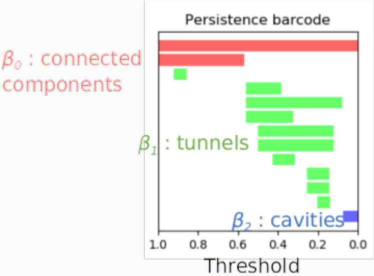
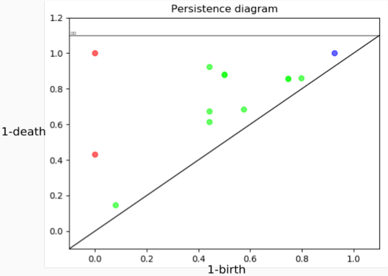


- Convert a gray image to a simplicial (cubical) complex.
- Calculate its PH (persistent barcode and diagram).





Topological loss function (Clough et al., 2020)

Assuming that the first β_k^* long bars are the "correct" ones (topological prior), the topological loss function is defined by

$$\mathcal{L}_{\text{topo}}(\beta_0^\star, \beta_1^\star, \beta_2^\star) = \sum_{k\in\{0,1,2\}} \left\{ \lambda_k^* \sum_{l=1}^{\beta_k^*} \left(1 - |b_{k,l} - d_{k,l}|^2\right) + \lambda_k \sum_{l=\beta_k^*+1}^{\infty} |b_{k,l} - d_{k,l}|^2 \right\}$$

where λ_k^* and λ_k are weights.

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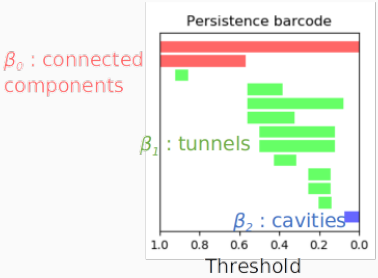
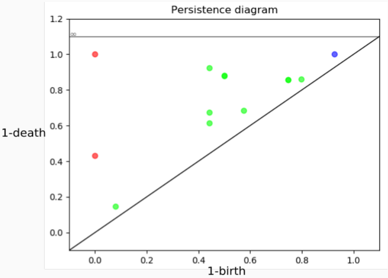
- **Elements to conserve:** move to $(0,1)$.





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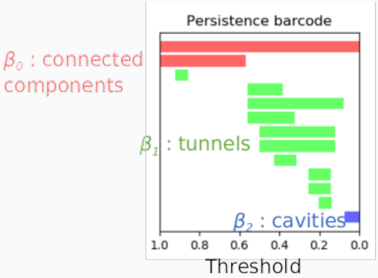
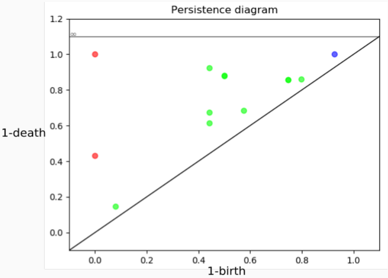
where λ_k^* and λ_k are weights.

- **Elements to conserve:** move to $(0,1)$.
- **Elements to eliminate:** move to the diagonal line.

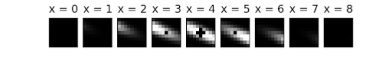


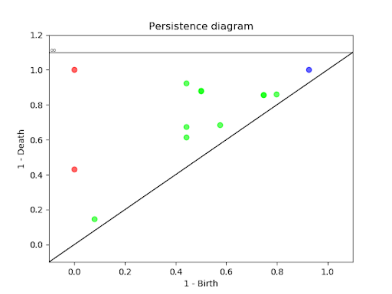


Topological prior $\mathcal{L}_{\text{topo}}(1, 0, 0)$

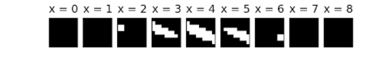
Setting $(\beta_0^*, \beta_1^*, \beta_2^*) = (1, 0, 0)$ favors one connected component.

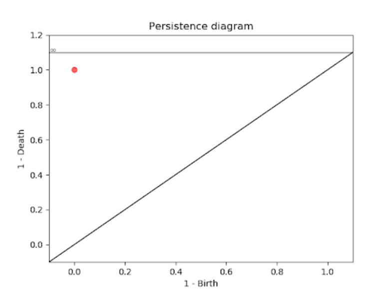
Input





Ideal reconstruction



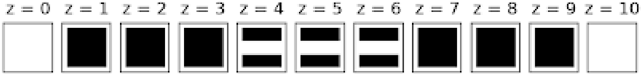


Topological priors $\mathcal{L}^{\square}_{\text{topo}}(1,1,1)$ and $\mathcal{L}^{\square}_{\text{topo}}(1,2,1)$

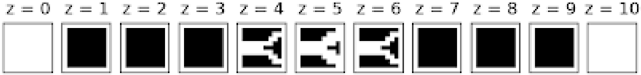
In order to distinguish a single tube and a bifurcation structure, we consider the **padded volume patches with a one-voxel border of value 1** and set

- $(\beta_0^*, \beta_1^*, \beta_2^*) = (1,1,1)$ for a single **tube** structure,
- $(\beta_0^*, \beta_1^*, \beta_2^*) = (1,2,1)$ for a single **bifurcation** structure.

Without bifurcation: Betti numbers $(1,1,1)$



With bifurcation: Betti numbers $(1,2,1)$



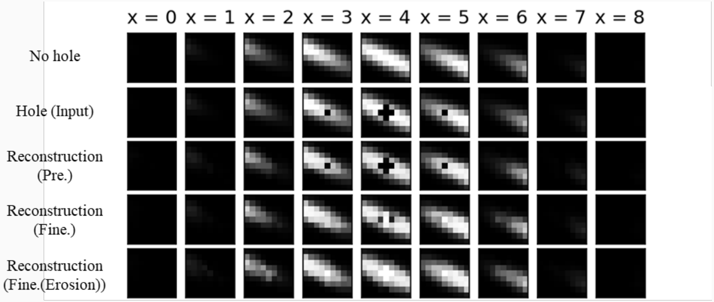
Experiment: blood vessels with hole artifacts

The network was pre-trained with only the β-VAE loss and fine-tuned by adding the **topological prior** of one connected component $\mathcal{L}_{\textbf{topo}}(1, 0, 0)$.

- **Datasets:** artificial volume patches (size $9 \times 9 \times 9$) of blood vessels with hole artifacts are generated; 6000, 2000 and 2000 are used for training, validation and testing.
- **Other setting:** the latent dimensions $J = 6$; the weights of the loss function $\beta = 0.1$, $\lambda_k^* = \lambda_k = 60000$.
- **Evaluation strategies:**
    - for intensity, **generalization** and **specificity** (Styner, 2003) were used,
    - for topology, the **topological loss function** with $\lambda_k^* = \lambda_k = 1$ was used.

Experiment: blood vessels with hole artifacts (results)

Given original volume patches without/with hole, compare its reconstruction without/with topological prior (Pre/Fine), and with the multiscale version (Erosion).



Evaluations

	Generalization	Specificity	Topology
Pre	**0.00251**	**0.0134**	0.430
Fine	0.00478	0.0143	0.0976
Fine + Erosion	0.00728	0.0156	**0.00139**

Experiment: blood vessels with bifurcations

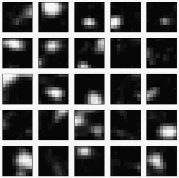
The network was pre-trained with only the β-VAE loss and fine-tuned by adding topological prior of bifurcation case $\mathcal{L}_{\text{topo}}^{\square}(1, 2, 1)$.

Dataset: volume patches of Y-shaped blood vessel

- CT images taken at Tokushima University Hospital (47 cases)
- thickness is approximately from 1 to 4 mm
- blood vessels are located between hilar and peripheral regions
- patches of size $9 \times 9 \times 9$ are extracted
  - after applying Hessian filter,
  - around local maximum of voxel values,
  - s.t. the PHs must be close to the topological prior.

1533, 517 and 517 volume patches are used for training, validation and testing.

Sagittal sections of 25 volume patches

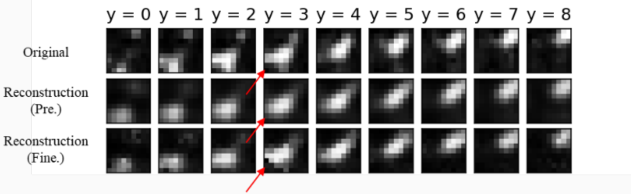


Experiment: blood vessels with bifurcations (cont.)

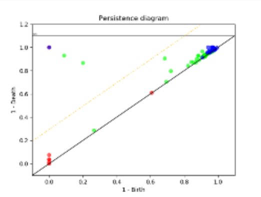
- **Other setting:** The latent dimensions $J = 22$; The weights of the loss function $\beta = 0.1$ and $\lambda_k^* = \lambda_k = 50$.
- **Evaluation strategies:**
  - for intensity, generalization and specificity (Styner, 2003) were used,
  - for topology, the Bottleneck distance between the PDs of the original and reconstructed images was used.

Experiment: blood vessels with bifurcations (results)

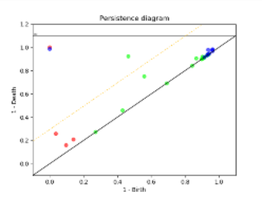
Given an original volume containing a bifurcation, compare its reconstruction without/with topological prior (Pre/Fine).



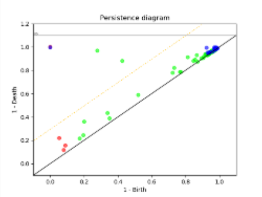
Original



Pre.



Fine



Evaluations

	Generalization	Specificity	Bottleneck distance
Pre	**0.00704**	**0.0283**	0.288
Fine	0.0102	0.0410	**0.236**

Conclusions and Perspectives

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- In order to incorporate topological priors of bifurcations into statistical modeling, we integrated topological loss into the deep generative model (β-VAE).
- Topological evaluation of reconstructed images was improved, and the improvement was also qualitatively identified.

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Perspectives

- How to guarantee topological prior?
- Apply the model to other tasks such as segmentation and super-resolution.